Institute of Actuaries of Australia

THE INSTITUTE OF ACTUARIES OF AUSTRALIA

Biennial Convention

May 2005

Modelling Risk:

Right Plan + Wrong Doing = Wrong Answer

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Presented to the Institute of Actuaries of Australia
2005 Biennial Convention 8 May – 11 May 2005

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Abstract

Actuaries, amongst others, spend a lot of time building models. It has been suggested by MacDonald, 1997, that 'the future of the actuarial profession is a matter of modelling and putting models into practice'. Models are used to assist with making decisions and to gain insight into the magnitude and behaviour of various risks. The use of a model does not remove the risks, but is intended to assist with their management. A key operational risk in using models is that the model, its data, its outputs, or the interpretations made of its outputs are flawed. Confidence in the management of such model risk is of key importance to users and decision makers.

We begin by examining what a model is and the components of the modelling process. A modelling process will usually include computations, but there are other elements of the process 'chain' that are equally important to the process assisting and improving expected outcomes.

Models, and their computational outputs, are used throughout the financial world in a wide variety of roles. We discuss some of the issues involved with models and computational choices and the consequent limitations that such choices may lead to. In practice, a balance between the theoretical and the practical will need to be made. If the data to support the appropriate choice of modelling parameters is not available, or the data to be processed is not reliable, then results may be flawed no matter how good the theoretical structure of the model may be. Also, in a commercial world, it is often the case that there is insufficient time to build a 'best' model, so compromises to reflect the circumstances are made. The impact of such compromises need to be controlled and understood. Alternatively, if a model is purchased, it still needs to be validated and understood before it can be used.

We then discuss some issues around the management of the modelling process. In some cases these are internally driven, but they may also be externally driven (at least to the extent of seeking demonstration of meeting minimal prescribed standards). There is a growing role, sometimes driven by regulatory requirements, for ongoing and external model validation and certification.

Key words: governance, computation, models, modelling, model risk, operational risk, risk management

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1 Introduction

This paper seeks to raise the awareness level of the risks in building and using models. We believe a better realisation and understanding of the range and potential size of the many risks involved should lead to better management and ultimately mitigation of these risks. The result should be an improvement in the likelihood of a more accurate result and/or a better understanding of the underlying situation being modelled.

"Right Plan + Wrong Doing = Wrong Answer" illustrates the basic tenet that no matter how good a model is, if it is not understood fully, or not used properly, the answers gleaned from it and so decisions made utilising these results, may be wrong or at the very least misleading.

Actuaries, amongst others, spend a lot of time building models. In various forms they are part of the tools of the actuarial trade. MacDonald, 1997, has suggested that 'the future of the actuarial profession is a matter of modelling and putting models into practice'. This puts models and the modelling process at the very core of actuarial practice. Models are used by decision makers to assist with making decisions and to gain insight into the magnitude and behaviour of various risks.

2 Models – Description and Use

2.1 What is a Model?

A definition of a "model" in the Australian Pocket Oxford Dictionary most likely to garner recognition by an actuary is:

Simplified description of a system etc to assist calculations and predictions

Extending this definition by replacing description with "representation" or "mechanism" gives us a better starting point for objectively assessing the use of models and the risks attaching to that use.

The simplification of a system (system is taken in its widest sense), requires the processes of the underlying mechanism to be formalised. In order to assess the applicability and validity of the model it needs to give results that can be consistently reproduced. Those results are then shared among the stakeholders and other interested parties.

How far does a model get from reality? That will depend on a range of factors including, the assumptions made in building it and in the data input to it, the extent of the simplifications, and the depth of the supporting theory. At the end of the day the proof of the pudding is in the eating – how close are the results of the model to the results from the actual process or mechanism being modeled.

2.2 Why Have Models?

Models are tools we use to assist with the management of future outcomes. In our case, these are usually financial services business outcomes. They are a means to an end rather than an end in themselves. They illustrate future uncertain events based on a range of conditions. They enable us to test decisions and see what reactions in a financial or other outcome might be to those decisions.

If we accept that the core specialist ability of actuaries is "making financial sense of the future" as the UK profession has summarised our offering to the world, then a corollary might be 'The future of the actuarial profession is a matter of putting models into practice' as this is a key tool in assessing potential future financial outcomes. Models are a natural outworking of attempting to take a proactive stance in the management of future outcomes, in contrast to a passive approach of sitting back and waiting for events to unfold.

2.3 Paradigms

Models used by professionals and the way they are used, tend to reflect current paradigms – i.e. current ways of thinking and tackling issues. This can be limiting, and it is important to be aware and seek to avoid such limitations.

By virtue of our education and experience there is a natural tendency to adopt approaches that have been used previously (presumably with success). As famously summarised by Jewell, 1980, this reflects the 'normal science' stage in Kuhn's 1970 model of scientific revolutions.

Most starting points for design and development of a new model is what we might term "Current 'best estimate.'" There is always the risk that this will reflect entrenched or vested interests and perspectives. For example, the use of models and approaches consistent with prior work may have its value, but also tends to blinker the developer to new ideas and approaches. In some cases there is an established 'industry' built around the use of specific models or approaches, and changes to this may be perceived as threats to the established order and power structures. To follow Kuhn again, this normal science paradigm usually dominates for a period until sufficient puzzles or crises arise within the paradigm that a revolution occurs and the old paradigm is overthrown for a new one. This new paradigm then settles and becomes accepted and an enhanced level of results and understanding is obtained, until it too falls in a repeat of the cycle. Perhaps a current example of this is the ongoing introduction of capital requirements for Operational Risk occurring in the Banking industry under the Basel II requirements.

The lesson for potential users of a specific model is to first ask the question "Is the model itself appropriate or is it dependent on too narrow an interpretation of what underlying situation variations might arise **-** if so, how else can the issue be approached?" There may be value in stepping back from the detail of both the current issue and prior approaches to similar problems and making an open minded review of the circumstances and options available.

2.4 Model Characterisation

Most models can be considered as comprising a number of "characterisations".

Firstly there is the generic issue to address. This requires the determination of a range of assumptions around the issue. Some of these will be obvious and some not. Consider what will affect the situation being modeled.

Most models and actuarial models require the setting of mathematical and/or logical relationships, particularly where financial transactions and their timing and value are being modeled. Theory will be used to set these "rules of engagement."

The computations and/or defined processes are carried out according to these rules and results are produced. The implementation of the theoretical computation may be a significant exercise in its own right, and can require specialised skills and tools.

To carry out these computations or processes, inputs are made to the model by way of parameters to reflect a set of assumptions. These parameters are often themselves derived from other underlying models, and are statistical inferences made that are based on reality. For example, the derivation of economic or demographic assumptions for a valuation model.

Finally, a model needs its base data to operate on - the fodder for the model to chew. For example, the provision of policy data for a valuation model. We draw a distinction between the parameters and data used in models as the issues and criteria used in their development and management are quite different. For instances, in some cases parameters may be mandated by external or regulatory constraints.

2.5 Modelling Process

We now draw a distinction between the computational steps in a model – producing some outputs – and the overall context or process within which models reside. The development and use of models is something that happens as part of a broader business environment and contributes to decision making and managing ongoing business processes. It is at this point the models developed to address generic problem are required to focus on the specifics of a given situation (perhaps otherwise known as customisation!).

The main steps in this broader modelling process are:

- Identify: Specific scope and purpose
- Approach: Choose model (in 'budget')
- Implement: Build, reliances (sub-models)
- Assess: Valid, robust, stable
- Parameters: Instances are link to reality
- Data: Useable and valid actual data
- Compute: Specific results
- Interpret: Messages and limitations
- Communicate: Consequences and options
- Decision: Users / owners

In particular note that the initial and later steps do not depend on the technical aspects of the model or the underlying theory.

2.6 Model Management

In a practical context, even before a modelling process is undertaken, business constraints are usually imposed. Like any process it needs to be managed, directed and held accountable in the larger context of business direction and priorities.

Some questions should be asked when considering the design of a new model.

- Intended use. What will it be used for? What is the problem and how will it assist in the resolution of this problem.
- Cost/ benefit: Closely related to this is the cost / benefit trade-off – in terms of priorities, resources, expertise available, costs and timelines.

- One-off or ongoing: In the longer term, is there an initial development phase with the intention to improve and increasingly use the model on an ongoing basis? Or is it just a "one-off" need as opposed to regular repeated use?
- Gains: What will be gained from the use of the results from the model. How do they illuminate the underlying issue being investigated.

Critical to ongoing maximisation of benefit from use is the continual refinement of the model and the use of the result generated. There is little point in developing a model and producing results if the analysis or interpretation of these results is poor or is not appropriately understood by or communicated to the ultimate users or decisions makers.

A control–cycle approach to the management of models should be used as this provides a mechanism to drive the accountability of actual results verses modelled results.

2.7 Chain for Model Use

We summarise the above by using the analogy of a chain. A chain is only as strong as its weakest link. A modelling process failure will result from failure of the weakest link in a long and often intricate and delicate chain involved in getting from the initial problem to a final decision. The key point here is that while the computational engine is a critical link in the chain, it is not the only link. The main links are set out below with an analogy to the design and development of a car and its use to reach a destination safely.

Model	*Car*
Theory	(designers)
Computational engine	(mechanics)
Parameters	(petrol, brake fluid …)
Data	(passengers, destination …)
Model Usage	(owners and car drivers)
Control – changes, new apps	(roadworthy)
Interpretation	(how to drive)
Communication	(driving)
Decision	(arriving – safely)

Failure in any of the car steps means you don't get there! Models carry greater risks – you may not arrive at the correct destination, but you may think you have!

Another way of thinking of this is that the model process chain highlights the importance of the 'use of the tool' rather then the importance of the tool (model) itself.

2.8 Non-Mathematical Models

Actuarial work tends to involve mathematical models. Most relate to the modelling of financial outcomes. However, we should not limit our thinking to computational models.

Examples of other models are Corporate Governance Models, such as that provided by the ASX, and other business models. Implementation of such models must cover external paradigms such as the law, and regulations and internal paradigms such as company culture and process.

Perhaps close to home, there is the Actuarial 'Control Cycle'. In the wider sense of a control cycle, it can be applied in a similar way to the further development of such models. Of course, there is nothing intrinsically actuarial about this approach – just a discipline to assess actual outcomes against what the model expected based on its underlying basis as "programmed" based on previous best estimate knowledge. The concept of control cycles and feedback loops has been established in many disciplines for many years.

From the professional actuarial perspective the control cycle concept is a powerful tool as it explicitly drives an accountable and positive professional culture.

2.9 Actuarial Capabilities

To reinforce the point that there is more to a modelling process – or chain **-** than the technical skills required in order to generate results, we list a set of attributes identified by our Institute in IAAust 2001, which were considered to be reasonable expectations of a qualified Fellow (after some years experience):

- Cognitive
- Expertise
- Actuarial Judgement
- Innovative and Flexible
- Rigour and Holistic
- Strategic
- Integrity
- Personal Management
- Influence and Interpersonal Skills
- Communication
- Business Acumen

The key observation on this list of attributes is the number of them which are not 'technically' oriented. It is these 'softer' attributes which are equally crucial in navigating the length of the chain for model management and allowing an actuary to move from the "right plan" though "right doing" to get the "right answer" using models.

3 Model Risks

3.1 "Obvious" Model Risks

When most people think of a model risk, it tends to be the more spectacular, headline cases.

A good example is the U.S. based HomeSide mortgage company errors that led to losses of around $AUS4bn in 2001 is a good example, see HomeSide, 2001. While there were many other things going on in this matter, it can be noted that a specific spreadsheet error was a significant contributor to the total loss.

More recently, both in Australia and the United States, there have been major losses attributed to unit pricing errors. Unit prices are computed using models – and in some cases the mathematical background to these models is considered to be relatively straightforward. This, however, does not seem to prevent the outcomes – the results provided at the end of modelling process chain – from being in error on occasion!

These remind us rather emphatically that errors do happen and the consequences are large and far reaching in financial and other (e.g. reputation) ways.

Most known errors fall into the implementation category – not the theory. Usually the theory is established and agreed – if not always in specific detail. Implementation then becomes the key. While often nowhere near as much fun as the higher level theoretical discussions, it is none the less critically important and is generally time consuming, requires careful attention to detail, precision, and clarity of scope. It would be rare for a model to be scoped initially and then not have changes and revisions made due to issues that emerge during the implementation and testing. A control cycle approach is warranted.

Problems that emerge include structural error or omissions, coding errors, use of wrong parameters, faulty data, misuse or misinterpretation of data.

To continue the car to model analogy, well maintained cars don't cause road carnage, the drivers do. To put this another way, it may be that the issue at hand is not the tool - the model - but the use of the tool - the use of the results obtained from the model – that is the issue. It is the result that emerges from the end of the modelling process chain that counts!

3.2 "Subtle" Model Risks

It is also important that the limitations of models be understood and communicated. It can sometimes be a hard question to identify and then communicate the circumstances in which a model, or more particularly the results it produces, should not be trusted.

The reasons for model limitations can be many and varied. There are the obvious ones of imprecision in input parameters or data. However there may also be more subtle issues to do with the limitations of the underlying theory or the instability of the theory in the face of imprecise data or parameters.

A good example of this later point is the Windcliff and Boyle, 2004, discussion of the '1/n Pension Investment Puzzle'. This paper examines the 'naïve' 1/n asset allocation for an investment portfolio. Without disagreeing with the standard theoretical approach of Markowitz efficient portfolios, the paper argues that the 1/n asset allocation remains consistent with the underlying theory once the practical issue of recognising there is limited information available from which to estimate the parameters. That is the theoretical model can be limited due to the uncertainty in the practical parameter uncertainty. In a sense the theoretical model is potentially 'unstable', and in this example a simpler (and apparently naïve) investment strategy is not as silly as it might first appear.

Consequently it is important to understanding what a model can do is to understand what it cannot do and should not be expected to do. The user must become aware and remain vigilant of the limitations of the model itself and understand how the results from the model might mislead in certain circumstances. These include:

- Extreme values of parameters
- Granularity. The model may only be able to get to a certain level of accuracy in its results. This limit needs to be understood and the model should not then be pushed to try to provide results that are more accurate that it can provide. Just because many digits in a result are output from a model, it does not follow that they are all reliable!
- Extreme values from full statistical distributions
- Underlying assumptions violated

- A change in the purpose of question (now) being asked of the modelling process relative to that for which it was developed.

3.3 Data Limitations

A good point for data limitations is the well-known modelling acronym GIGO: Garbage In - Garbage Out! General Insurance practitioners when assessing outstanding claims will be particularly aware of the effect this had on HIH which in turn led to the Royal Commission, HIH 2003, to make strong recommendations for the need to validate the data used in actuarial valuation models.

Sufficient time needs to be spent to validate all input data to a model to ensure its integrity and fitness for the modelling purpose at hand, given an understanding of how the model will use the data, including any implicit and explicit assumptions in that use.

3.4 Modelling Errors are NOT Infrequent

Modelling errors can arise from one or more of the obvious or subtle risks, or data errors or limitations described above. How common are they? How many are known widely and how many are kept to as restricted a range of people as possible to avoid embarrassment or worse? How serious are they?

To start with, one can form a view from one's own direct experience in using someone else's model and finding something "not quite right" when using it for the first time. Another insight is from the ongoing litany of issues that emerge in the press regarding model and (often) spreadsheet errors. To get a view on the pervasiveness of these errors, one merely has to 'do a google' and search for 'error spreadsheet' – and then refine the 800,00 references that come up (when we did this).

An attempt to quantify the error rates in spreadsheets was made by Pryor, 2003. The results are attention getting in that they report a number of surveys with the headline result suggesting that the majority of spreadsheets are likely to contain errors. Obviously as the complexity of spreadsheets grows the likelihood of errors grows. However, most of us can probably think of fairly basic spreadsheets in which errors have occurred as well. There is also evidence quoted by Pryor to suggest that people are overly confident in their capabilities to avoid making errors (and so potentially do not check their work as carefully as might otherwise be the case). With large spreadsheets and other models of potentially greater complexity commonly in use, there seems to be little space for complacency in terms of addressing modelling errors. We note that modelling errors (that is, errors at the end of the model process chain) include data and parameter errors as well as theoretical and coding errors. The use of models under time and resource constraints is also a source of error.

While there are guides and methodologies around intended to assist with the building of models, see for example, Raffensperger, 2000. However they often provide the form rather than the content of the task. That is not to say that they are not useful guides, as they are. However it is to say that the key to good model building lies in the intent and understanding and not in the tools used.

As noted earlier, a parallel can be drawn with unit pricing errors which are a major focus of the wealth management industry, APRA and ASIC at present. The calculations may be conceptually simple but the full process can be complex and error prone.

There is also a danger in businesses using models to place too much reliance on 'the expert' because they have relatively more knowledge or understanding of the model. The "Trust me, I built it …" approach is dangerous.

The best counter to this is the application of experience, intuition and a healthy skepticism with arms length "smell tests".

3.5 Operational Risk

APRA and The Bank for International Settlements (BIS) in Basel defines Operational Risk as:

> *The risk of loss resulting from inadequate or failed internal processes, people or systems or from external events.*

See BCBS, 2004.

Most model risks can be considered as a subset of operational risks.

However, we emphasise the issues around the risks of implementation, which we term Application Risk. We suggest that Application Risk is a key element of Operational Risk in general as well as in the specific context of models

Application Risk is the risk of "*Doing it wrong'* in contrast to "*Knowing what to do*". To paraphrase, it is the distinction between theory and practice. We suggest that the theory may be something like 5% of the game and the implementation the remaining 95% - recall the famous saying that "g*enius is 5% inspiration and 95% perspiration"*. Alternatively, think of how many good ideas you have had or heard about but which never come to fruition!

To emphasise the importance of Application Risk, consider the following simple analogy: Everyone knows the theory of how to join two pieces of wood using a hammer and a nail. It is perhaps a very different issue to be given a set of pieces of expensive custom dressed wood, an array of hammers and a wide variety of nails (and perhaps glues to help), and then told to provide a professional quality structure (in accord with a theoretical plan) from these pieces of wood for an important and demanding client. The distinction between the theory and the implementation, for most of us, would be abundantly clear in the result of this exercise!

4 Model Management and Application

4.1 Risk Mitigation

As with most standard risk management approaches, the identification of the possible range and size of risks that might be present in or related to use of a model, is the first step in mitigating those risks. In addition to the fundamental of raising one's awareness level and hence channeling this into all uses and assessment of the model, the following actions are recommended to reduce the risk of the model or its use giving or leading to misleading or incorrect results and ultimately wrong business decisions.

Model validation
This extends to all aspects – including design, methodology, technical correctness and reasonableness of results. Initially this should be done internally as part of the model's development and thereafter as part of change control. In addition, independent third party

validation is the best way of reviewing. The more specialist and/or complicated the model is, the more important it becomes to validate externally.

Such validation extends beyond an audit or simple compliance with regulatory or legal requirements. The latter should be considered as minimal standard rather than a marker of 'good practice'.

Complexity
Models are often so complex that it is simply not possible or practical to test all computational paths they may contain. This implies users should never treat the computational model they have as a 'black box' from which will emerge a 'correct' solution. That is the ownership of the results and their validity ultimately should always reside with the owner of the model and not be attributed to the model. This in turn implies the cultural need for this ownership and accountability to be taken on and acknowledged at each step through the modelling process chain.

Reality recognition
As with most processes and in particular with long or complicated process, it is important to be realistic and recognize that errors WILL happen from time to time. Risk mitigation is achieved by establishing processes to catch, "cap" and fix these errors, *before* they "leave the modelling room".

Other
Having clear responsibility for and taking ownership of the results separate from responsibility for the process itself is important. In general terms, understanding the limitations of the model and being able to communicate these to those interpreting the results are critical.

4.2 Organisational Culture

Many of the attributes required to successfully use models rely on the integrity and commitment of the users. These are essentially cultural attributes, and they are moulded by the culture of organisations. The importance of organisational culture in general, and in the management of models and their results in particular, is hard to over-emphasise. The test of these cultural norms is when things go wrong. In your organisation, reflect on whether the norm when an error occurs, or may be occurring, is to highlight the issue in an open way and then get on with the remedy in a no blame atmosphere, or is it to fix it quickly and then present a solution as a 'fait accompli' before anyone else finds out?

As an aside, recent work by MacDonald, 2002, in examining failures and near failures of insurers identified the primary cause of failure as poor management. Cultural values are driven 'from the top'. This work found other 'reasons' are symptoms rather than causes. The parallel for modelling, is that errors or failures in models are not the fault of the model, but rather that of poor use of the result from the model by its users. The issue is primarily of organisational culture rather than technical accuracy.

4.3 Actuarial (Modelling) Practice

We can put a process around the modelling process. The following schematic is based on Gribble, 2003.

Actuarial (Modelling) Practice

Problem:
    Computational Tool
    + Analytic Cycle
        Specify
        Solve
        Monitor
    + Professional Cycle
        Professionalism
        Risk
        Implementation
        Environment
= Modelling Process
    + Application to Financial Services
= Model Management
    + Actuarial Capabilities
= Actuarial (Modelling) Practice

Having established a modelling process, it needs to be actively managed in its application to ensure it is used correctly and the risks of errors or misuse are minimized. Actuarial skills and experience can add value to each element of both the Analytic and Professional layer, but the challenge for the profession is often to rise above these technical issues and add value at the end of the modelling process chain, not just to some of the links in the chain.

We also note the basic similarity of the Analytic Cycle with the recently updated and internationally accepted AS/NZS 4360:2004 Standard on Risk Management, Standards Australia 2004.

Good actuarial practice in modelling can be summarised as the combination of the model characterisation, process and management all the way through to the actuarial practice of augmenting decision making.

5 Summary

The use of a model does not remove the risks, but is intended to assist with their management. A key operational risk in using models is that the model, its data, its outputs, or the interpretations made of its outputs are flawed. Confidence in the management of such model risk is of key importance to users and decision makers.

Having described the characteristics of models, the processes used to design and build them, and their use, we discussed the wide range and variable level of risks associated with them. We concluded:

- Most model risk arises from implementation, that is, Application Risk within Operational Risk
- Culture of an organisation is critical to reducing this risk – an open culture encourages active and ongoing constructive critical validation of models and their data inputs, and their correction and improvement.
- Errors / ‘failures’ will happen - business is a complex and changing environment. Companies should explicitly plan the actions to be taken speedily to rapidly correct the “errors” when they arise.

- There are ways to mitigate modelling and application risks
- A company's important models should be independently validated by an external modelling expert.

6 Acknowledgements

We thank Richard Donaldson for his helpful comments in reviewing this paper.

7 References

BCBS (Basel Committee on Banking Supervision) 2004, "*International Convergence of Capital Measurement and Capital Standards – A Revised Framework*", June 2004.

Gribble J D, 2003, "Actuarial Practice and Control: Objectives and Capabilities", University of Melbourne, Centre for Actuarial Studies Research Paper 105, 2003.

HIH, 2003, "Report of the HIH Royal Commission", see http://www.hihroyalcom.gov.au, 2003.

Homeside, 2001, http://www.crikey.com.au/articles/2001/12/02-nbhomeside.html

IAAust 2001, 'Actuarial Capabilities – Final Draft', Prepared by Learning Paradigms, 2001.

Jewell W S, 1980, "Generalised Models of the Insurance Business (lie and/or non-life insurance)", Transactions of the 21st International Congress of Actuaries, Vol S, 1980.

Kuhn T S, 1970, "The Structure of Scientific Revolutions", 2nd edition, University of Chicago Press, 1970.

MacDonald A S, 1997, "Current Actuarial Modelling Practice and Related Issues and Questions", North American Actuarial Journal, Vol 1, No 3, 1997.

MacDonnell W, 2002, "Managing Risk: Practical Lessons From Recent 'Failures' of EU Insurers", FSA Occasional Papers No 20, London, December 2002

Pryor, L, 2003, "Spreadsheet Error Rates", http://www.louisepryor.com/showTopic.do?topic=31, 2003.

Raffensperger J F, 2000, "The New Guidelines for Writing Spreadsheets", Department of Management, University of Canterbury, Christchurch, New Zealand, 2000.

Standards Australia 2004, "AS/NZS 4360:2004 Australia / New Zealand Standard: Risk Management", Standards Australia, 2004.

Windcliff H and Boyle P P, 2004, "The 1/n Pension Investment Puzzle", North American Actuarial Journal, Vol 8, No 3, 2004.